THE IMPACT OF FFT SIZE ON THE PERFORMANCE OF A COMBINED OFDM-EQUALIZATION RADIO MODEM

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ABSTRACT

Conventional OFDM systems employ a guard interval to combat delay spread distortion of transmitted data. This reduces the efficiency of the OFDM transmission. A combined OFDM-Equalization transmission strategy is presented in this paper. This strategy employs an Adaptive Equalizer to combat delay spread distortion instead of a guard interval. This facilitates the use of very short guard intervals and thus the efficiency of the OFDM transmission is improved. This paper presents the combined OFDM-equalization technique and demonstrates the efficiency improvement that it offers over conventional OFDM. The performance of the pre-FFT equalizer is sensitive to time variation of the mobile radio channel's impulse response. Software simulation is employed to analyze this sensitivity and a relationship is determined between performance and FFT size. The results are considered in the context of application to existing and future mobile radio systems.

I. INTRODUCTION

The technique of Orthogonal Frequency Division Multiplexing (OFDM) offers a robust method for digital radio transmission. OFDM is particularly suitable for mobile radio applications.

Conventionally, OFDM employs a guard interval to combat delay spread of the radio signal that will result in inter-carrier interference (ICI) of the OFDM modulated data. The use of a guard interval reduces transmission efficiency. In this paper, a combined OFDM-equalization strategy, incorporating a novel pre-FFT equalizer design [1] in the receiver, is proposed. This combined OFDM-equalization strategy allows for a considerable reduction in the length of the guard interval and thus offers a significant improvement in transmission efficiency over conventional OFDM.

The pre-FFT Equalizer directly cancels delay spread effects to enable the guard interval to be shortened. However, in order to enable decision directed adaptation of the pre-FFT equalizer, it is necessary to employ a modified version of the LMS algorithm for adaptation. This modified algorithm adapts at the OFDM symbol rate rather than the transmission symbol rate and as a result the pre-FFT equalizer suffers greater impairment under mobile conditions than would be the case for an equalizer in a single carrier system.

OFDM is the specified modulation method for the ETSI Terrestrial Digital Television Broadcast (DVB-T) [2] and High Performance Radio Local Area Network type 2 (HIPERLAN/2) standards. The suitability of the combined OFDM-equalization technique for these applications is considered in this paper.

This paper presents the OFDM modulation process in section II to provide mathematical reference for the later sections. The combined OFDM-equalization receiver is presented in section III and the pre-FFT equalizer is described in section IV. In section V the efficiency of both conventional and combined techniques are defined and compared. Software simulation has been undertaken to determine the performance of the pre-FFT equalizer design in the mobile channel. The results of this simulation are presented in section VI. The relationship between performance and FFT size is determined and discussed in the context of application to mobile radio. In particular, application to mobile DVB-T and HIPERLAN/2 are considered.

II. OFDM TRANSMISSION

A conventional OFDM modulation process [3] as illustrated in Figure 1 is employed to generate the transmitted signal for reception by both conventional OFDM and combined OFDM-

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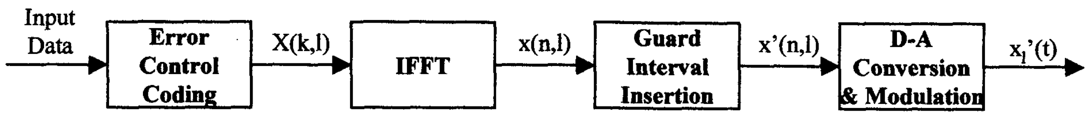

Figure 1. OFDM Modulation

equalization. A frequency domain input data vector, $X(k,l)$ consisting of N data symbols is input to an IFFT to produce a time domain vector, $x(n,l)$. (Here, k indexes the OFDM sub-band, n indexes the transmission symbol and l indexes the OFDM symbol.) The data symbol period is T_s and the OFDM symbol period is NT_s.

The time sequence is cyclically extended by M symbols to produce the transmission vector $x'(n,l)$.

$x'(n,l)$ is up-sampled, D-A converted and RF modulated to produce the transmission signal $x_l'(t)$.

III. THE COMBINED OFDM-EQUALIZATION RECEIVER

The structure of an OFDM receiver employing a pre-FFT Equalizer is shown in Figure 2. Its function can be seen to be that of a conventional OFDM receiver [3] with the addition of the adaptive equalizing filter and a feedback loop.

$y'(n,l)$ is applied to the equalizing filter and $z'(n,l)$ results. The M symbols of $z'(n,l)$ which represent the cyclic extension are discarded and an FFT is applied to produce $Z(k,l)$. $Z(k,l)$ may be applied to a channel compensator to produce $V(k,l)$. However, if the pre-FFT equalizer is suitably adapted: $V(k,l) \approx Z(k,l)$.

$V(k,l)$ is applied to a decision device to produce the output data $W(k,l)$.

The feedback path in the receiver generates $w'(n,l)$ which is an estimate of the transmitted OFDM symbol, $x'(n,l)$ based on the post-decision output data symbols.

IV. THE PRE-FFT EQUALIZER

The structure of the pre-FFT equalizer is shown in Figure 3. This design is similar to that of a Decision Feedback Equalizer (DFE) that might be used in a single carrier system [4]. An adaptation strategy combining the use of regular training sequences and decision directed adaptation can be employed. The equalizer output is defined as:

$$z'(n,l) = \sum_{j=-J_1}^{-J_1+n} c(j)y'((n-j-(N+M)),l+1) + \sum_{j=-J_1+n+1}^{0} c(j)y'((n-j),l) + C_{out}(n)\sum_{j=1}^{n} c(j)z'((n-j),l) + \sum_{j=n+1}^{J_2} c(j)w'((n+(N+M)-j),l-1) \tag{1}$$

where:

$$C_{out}(n) = 0 \quad \text{for} \quad n = 0$$
$$C_{out}(n) = 1 \quad \text{for} \quad n \neq 0 \tag{2}$$

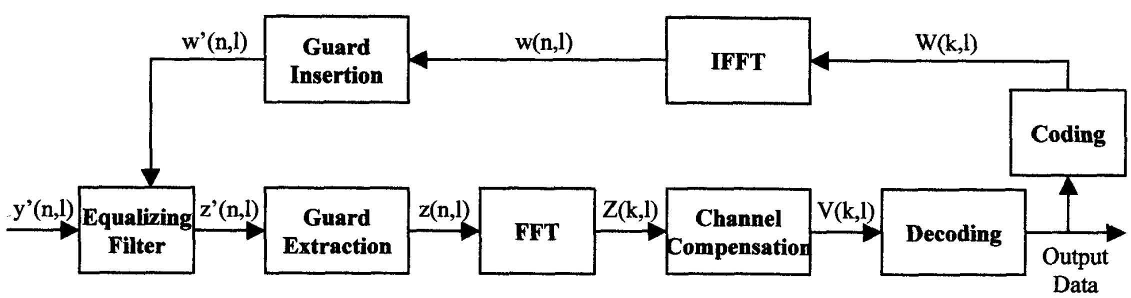

Figure 2. Combined OFDM-Equalization Receiver

J_1 and J_2 respectively define the lengths of the feedforward and feedback sections of the equalizer.

Equalizer Training can be implemented in a conventional manner, such as by the LMS algorithm [5]. The LMS algorithm for the pre-FFT equalizer is described by:

$$c(j,n+1,l)=c(j,n,l)+\Delta\varepsilon'(n,l)y'^{*}((n-j-(N+M)),l+1)$$
$$\text{for } -J_1\le j\le -J_1+n+1 \qquad (3)$$

$$c(j,n+1,l)=c(j,n,l)+\Delta\varepsilon'(n,l)y'^{*}((n-j),l)$$
$$\text{for } -J_1+n+2\le j\le 0 \qquad (4)$$

$$c(j,n+1,l)=c(j,n,l)+\Delta\varepsilon'(n,l)x'^{*}((n-j),l)$$
$$\text{for } 1\le j\le n \qquad (5)$$

$$c(j,n+1,l)=c(j,n,l)+\Delta\varepsilon'(n,l)x'^{*}((n+(N+M)-j),l-1)$$
$$\text{for } n+1\le j\le J_2 \qquad (6)$$

$$\varepsilon'(n,l)=x'(n,l)-z'(n,l) \qquad (7)$$

Decision Directed Adaptation is made more complex by the parallel transmission nature of OFDM. Since a complete OFDM symbol must be received before it can be processed by the FFT and the decision device, the equalizer can only be updated at intervals of the OFDM symbol period [5]. Thus, at intervals of the OFDM symbol period the equalizer is adapted according to all the transmission sub-symbols for the preceding OFDM symbol. Also, it is necessary to input pre-decision symbols into the equalizer's feedback section until a complete OFDM symbol has been received and fed back to the equalizer. The LMS algorithm can be modified to adapt the equalizer in decision directed fashion and is now described by:

$$c(j,n,l+1)=c(j,n,l)+C_{ff}(j)\sum_{n=0}^{N+M+j-1}\Delta\varepsilon'(n,l)y'((n-j),l)+\sum_{n=N+M+j}^{N+M-1}\Delta\varepsilon'(n,l)y'((n-(N+M)-j),l+1)$$
$$\text{for } -J_1\le j\le 0 \qquad (8)$$

$$c(j,n,l+1)=c(j,n,l)+\sum_{n=0}^{j-1}\Delta\varepsilon'(n,l)w'((n+(N+M)-j),l-1)+C_{fb}(j)\sum_{n=j}^{N+M-1}\Delta\varepsilon'(n,l)w'((n-j),l)$$
$$\text{for } 1\le j\le J_2 \qquad (9)$$

$$\varepsilon'(n,l)=w'(n,l)-z'(n,l) \qquad (10)$$

where:

$$C_{ff}(j)=0 \quad \text{for} \quad -j=N+M$$
$$C_{ff}(j)=1 \quad \text{for} \quad -j\ne N+M \qquad (11)$$

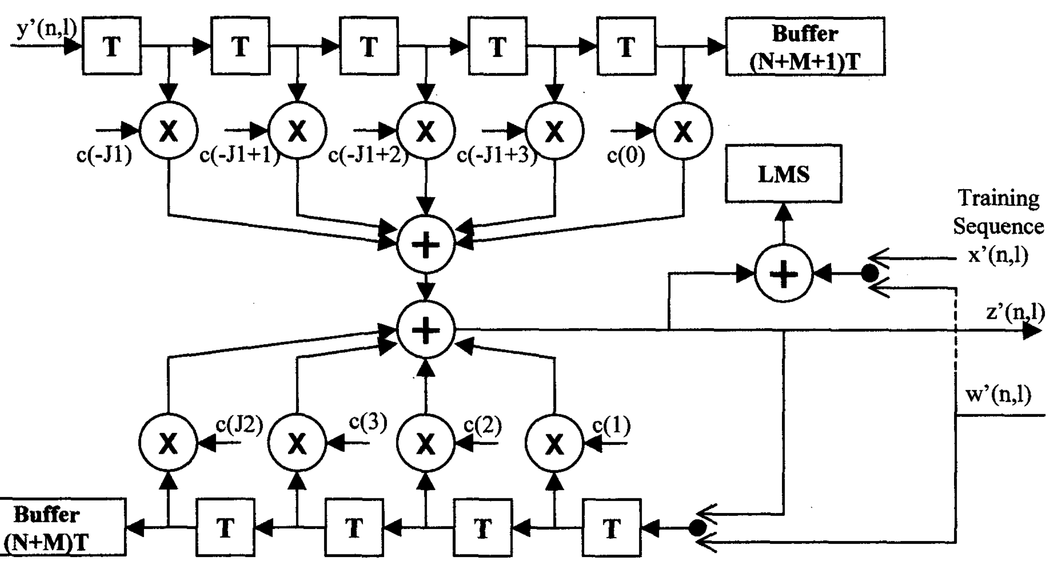

Figure 3. Pre-FFT Equalizer

$$C_{fb}(j)=0 \quad \text{for} \quad j=N+M$$
$$C_{fb}(j)=1 \quad \text{for} \quad j \neq N+M \qquad (12)$$

V. TRANSMISSION EFFICIENCY

The efficiency of the OFDM modulation scheme is defined as:

$$\varepsilon_{MOD}=\frac{N}{N+M} \qquad (13)$$

For conventional OFDM, the condition to prevent ICI is:

$$MT_s > \tau_{MAX+} + \tau_{MAX-} \qquad (14)$$

where τ_{MAX+} and τ_{MAX-} are the maximum delay and advance of the radio channel. Thus the transmission efficiency of conventional OFDM is limited according to:

$$\varepsilon_{CONV} < \frac{NT_s}{NT_s+(\tau_{MAX+}+\tau_{MAX-})} \qquad (15)$$

For combined OFDM-equalization, provided the equalizer is suitably adapted, the condition to prevent ICI is:

$$J_2T_s > \tau_{MAX+} \qquad (16)$$

and:

$$J_1T_s > \tau_{MAX-} \qquad (17)$$

Thus, the efficiency of the combined OFDM-equalization technique is limited by the training requirements of the equalizer instead of the guard interval requirement. Consequently, if a training sequence is employed at regular intervals, the efficiency of the combined technique can be described by:

$$\varepsilon_{CONV}=\frac{L_{DD}}{L_{DD}+L_T} \qquad (18)$$

where L_T and L_{DD} are the number of symbols in the training sequence and data sequence respectively.

Typically, conventional OFDM systems can operate under conditions where the maximum delay spread is up to one quarter of the OFDM symbol period. Under such conditions $\varepsilon_{COMB}=80\%$. For the same delay spread, a combined OFDM-equalization strategy can operate with $L_T \le 2$ [5] and $L_{DD} \ge 100$ (see section VI). Thus, $\varepsilon_{COMB} \ge 98\%$ is possible.

VI. SIMULATION RESULTS

As described in section IV, the combined OFDM-equalization strategy employs a modified version of the LMS algorithm to achieve decision directed adaptation. As a result, the performance of the equalizer is sensitive to any significant variations in the radio channel's impulse response that occurs within the OFDM symbol period. To assess this sensitivity, the equalizer's performance under mobile conditions was simulated in software.

The radio channel was modeled by a tap delay line filter. The tap coefficients were varied according to independent Rayleigh distributions with the time average of the coefficients over the total transmission time described by a Rician distribution. The rate of variation of the tap coefficients was determined according to the velocity of the receiver. The total delay period of the channel, as normalized to the transmission symbol period was, $d_{MAX}=7.68$, where:

$$d_{MAX}=\frac{\tau_{MAX+}+\tau_{MAX-}}{T_s} \qquad (19)$$

The MSE of the equalizer's output is shown in Figure 4, plotted against the normalized Doppler frequency, F_DT_s for various values of N. A QPSK symbol constellation was employed and for all cases, $J_1=J_2=N$, $L_T=5$, $L_{DD}=95$ and $\varepsilon_{COMB}\approx 95\%$.

The MSE results show a consistent trend with, as expected, a very low error for the static case ($F_dT_s=0$), a rapid rise in error as the channel becomes mobile and a slower rise in error as mobility increases. Eventually, the error curves reach a critical point, beyond which the MSE rises very rapidly. This critical point represents the maximum Doppler frequency that can be supported by the pre-FFT equalizer for a given value of N. Above this value there is a high probability of the equalizer diverging due to significant changes in the channel's impulse response occurring within one OFDM symbol period. From Figure 4 it can be seen that the maximum normalized Doppler frequency that can be supported by the pre-FFT equalizer is:

$$(F_DT_s)_{MAX}\approx\frac{0.0025}{N} \qquad (20)$$

Similar analysis has been undertaken in the past to assess the sensitivity of single carrier equalizer systems to mobile channel variation [6][7]. This analysis indicates that a single carrier equalizer can support a normalized Doppler frequency of approximately 0.003. Considering equation 20 and noting that $N=1$ for a single carrier system it can be seen that there is a strong correlation between the results presented in this section and those published elsewhere.

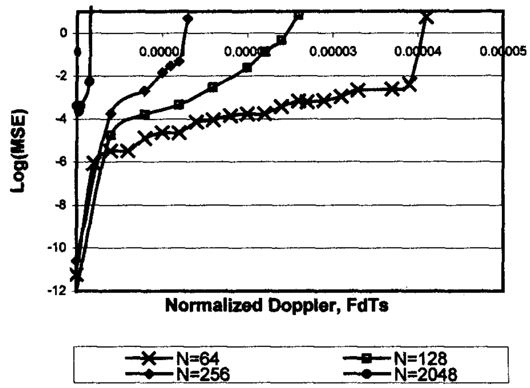

Figure 4. MSE vs. Normalized Doppler

Using equation 20 to determine the maximum normalized Doppler spread, it is possible to determine the maximum supportable receiver velocity, v_{MAX} from the transmission rate and:

$$v_{MAX} = \frac{F_D c}{F_T} \tag{21}$$

where F_T is the transmission frequency and c is the velocity of light.

For DVB-T, N is 2048 or 8192, $T_s \approx 100ns$ and F_T is between 470 and 850MHz. Thus the maximum receiver velocity which can be supported is 4.73ms^{-1} for $N = 2048$ and 1.18ms^{-1} for $N = 8192$.

HIPERLAN/2 proposals are for $N = 64$, $T_s = 50ns$, and $F_T \approx 5GHz$. In this case, the maximum receiver velocity is 49ms^{-1}.

VII. CONCLUSIONS

It has been shown that the combined OFDM-equalization technique can achieve significant improvements in transmission efficiency over the conventional OFDM technique. Improvements of up to 18% can be achieved in the case were delay spread is particularly severe. Further more, the efficiency of the combined OFDM-equalization technique is not dependent upon the FFT size employed as is the case for conventional OFDM.

It is clear from Equation 20 that the maximum receiver velocity under which a combined OFDM equalization receiver can operate is limited according to the inverse of the FFT size employed. For systems such as DVB-T, which use very large FFT sizes, application is limited to static and relatively slow moving receivers. This is unsuitable for many vehicular receivers. For systems such as HIPERLAN/2, which use much smaller FFT sizes, much higher velocities, suitable for most vehicular applications, can be accommodated.

Thus, the combined OFDM-equalization receiver can be applied to systems with small FFT sizes to achieve significant improvements in efficiency with no penalty other than the computational load required to perform the LMS adaptation. The combined OFDM-equalization receiver can also be used to improve the efficiency of OFDM systems with large FFT sizes but at the additional penalty of restricted rates of mobility.

However, since the efficiency of a system employing a combined OFDM-equalization receiver is independent of the FFT size, the technique will also allow future systems operating in similar scenarios to DVB-T to employ smaller FFTs with high efficiency and also operate under highly mobile conditions.

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ACKNOWLEDGEMENTS

The authors would like to thank News Digital Systems (NDS) for their financial support of this project. In particular, we would like to thank Arthur Mason OBE and his colleagues for their enthusiastic support and comments throughout the project.